**In-Class Exam #4 Review Sheet, Covers 11.10, 11.11, 10.1-10.4**
**Math 280, Vanden Eynden**

*In problems#1–5, find the Maclaurin series for f and its radius of convergence. To find each power series, you may use either the direct method (definition of a Maclaurin series, taking several derivatives and finding a pattern) or use known series such as geometric series, binomial series or the Maclaurin series shown in Section 11.10, Table 1, pg 762.*

1. $f(x) = x^2 e^{-3x}$

2. $f(x) = \dfrac{x - \sin x}{x^3}$

3. $f(x) = 2^x$

4. $f(x) = \sqrt[3]{1-x}$ (binomial series)

5. $f(x) = (1+x)^{-3}$ (binomial series)

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6. Use the Maclaurin series found in #2 to approximate $\displaystyle\int_{0.1}^{0.4} \frac{x - \sin x}{x^3}\, dx$ to within |error| < 0.001.

7. a. Find the degree 3 Taylor polynomial, $T_3(x)$ for the function $f(x) = e^{2x}$ centered at $a = -1$.

   b. Use your Taylor polynomial of degree 3 from above to approximate the value of $e^{-2.2} = e^{2(-1.1)}$. Round your approximation to 4 decimal places.

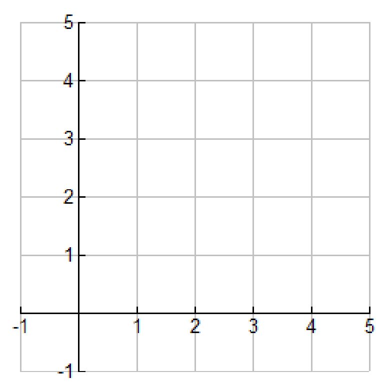
8. a. Find the degree 4 Taylor polynomial, $T_4(x)$ for the function $f(x) = \ln\left(x^2\right)$ centered at $a = 2$.

   b. Use your Taylor polynomial of degree 4 from above to approximate the value of $\ln(1.8^2)$. Round your approximation to 4 decimal places.

9. a. Fill in the table of values below for the given parametric equations. Then graph the curve, including arrows indicating direction of movement.

$x = t - 1 \qquad y = \dfrac{t}{t-1}$

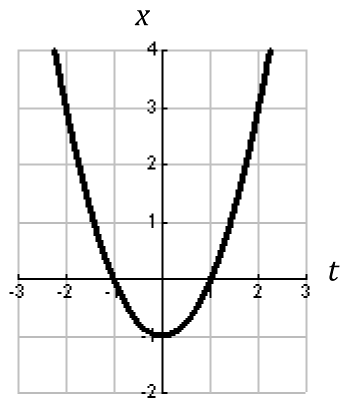
| t | x | y |
|---|---|---|
| 1.5 | | |
| 2 | | |
| 2.5 | | |
| 3 | | |
| 3.5 | | |
| 4 | | |
| 4.5 | | |
| 5 | | |

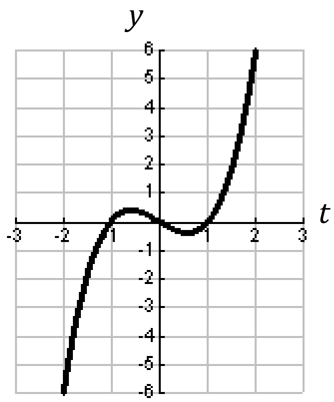



   b. Now, eliminate the parameter to obtain a Cartesian equation for the curve.

10. Find a polar equation for a circle centered at (0,0) with radius 3.

11. Use the graphs of $x = f(t)$ and $y = g(t)$ below to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate with arrows the direction in which the curve is traced as $t$ increases.




12. Find the equation of the tangent to the curve $x = t^3 + 6t + 1$, $y = 2t - t^2$ at $t = -1$

13. Use your graphing calculator to estimate the coordinates of the lowest point on the curve $x = t^3 - 3t, y = t^2 + t + 1$. Then use calculus to find the exact coordinates.

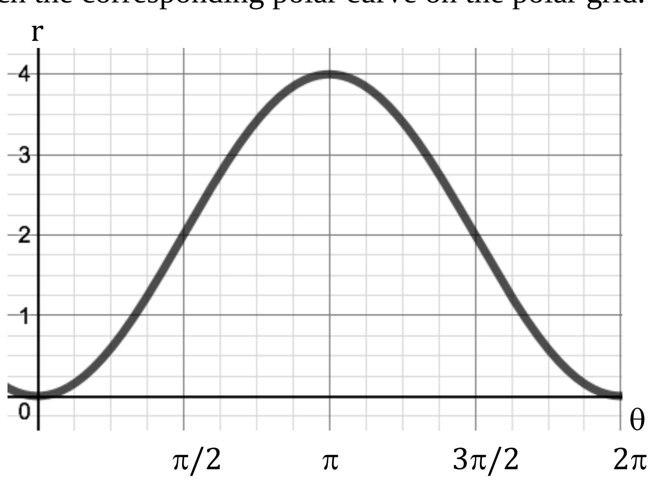
14. Find $\frac{d^2y}{dx^2}$ for the curve $x = t + \sin t$, $y = t - \cos t$. Is this curve concave up or concave down at the point (0,–1)?

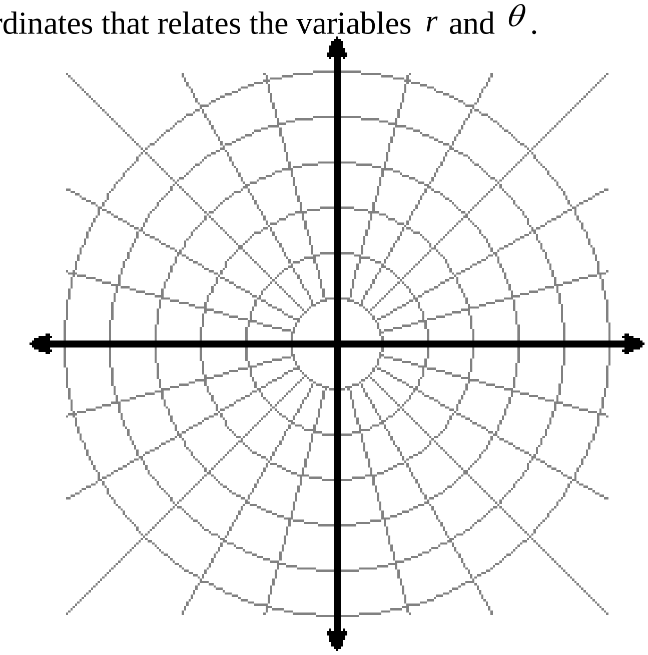
15. Over what interval(s) of $t$ is the parametric curve defined by $x = 1 + t^2$ $y = t - t^3$ concave downward?

16. Find the length of the curve defined by $x = 3t^2$, $y = 2t^3$ on the interval $0 \le t \le 2$.

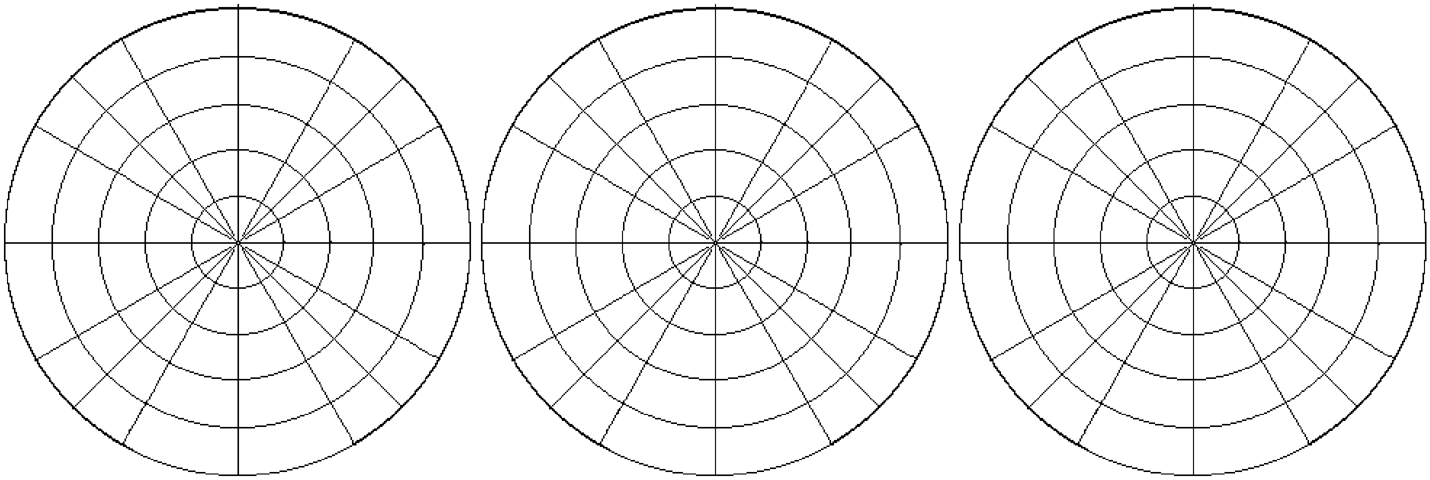
17. Consider the graph shown below in rectangular coordinates that relates the variables $r$ and $\theta$.

Sketch the corresponding polar curve on the polar grid.



*In problems 18, 19, 20 use the polar grids below to sketch your graphs. Be accurate!*



18. Sketch the polar curve $r = 3 + \cos 3\theta$. Use your calculator to help.

19. Sketch the polar curve $r = 5\sin 3\theta$. Use your calculator to help.
    What is this curve called? ________________________

20. Shade/sketch the region defined by $r > 2$ and $\pi \le \theta < \frac{5\pi}{4}$

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21. Find the points of intersection for the curves $r = 2$ and $r = 4\cos\theta$. State the points using polar coordinates. Then convert those same points to Cartesian coordinates $(x, y)$.

22. Find a Cartesian equation for the curve represented by the polar equation $\sin\theta + \cos\theta = r$.

23. Find the area enclosed by the inner loop of the curve $r = 1 - 2\sin\theta$.

24. Find the area of the region that lies inside both of the circles $r = 2\sin\theta$ and $r = \sin\theta + \cos\theta$.

25. Find the area of the region inside the large loop and outside the small loop of $r = 1 + 2\cos\theta$.
    Set up the integral(s) required to calculate this area ONLY. (you can use calculator or wolframalpha.com to find the actual area)

**Exam 4 Formula Sheet** **(You will be given fresh copy on exam day)**
**Covers 11.10, 11.11, 10.1-10.4**
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**Taylor series of the function *f* centered at *a*:** $$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

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**The Binomial Series:** If $k$ is any real number and $|x|<1$, then

$$(1+x)^k = \sum_{n=0}^{\infty}\binom{k}{n}x^n$$

$$=1+kx+\frac{k(k-1)}{2!}x^2+\frac{k(k-1)(k-2)}{3!}x^3+\frac{k(k-1)(k-2)(k-3)}{4!}x^4+\dots$$

where $\binom{k}{n}=\dfrac{k(k-1)(k-2)(k-3)\cdots(k-n+1)}{n!}$ are called binomial coefficients.

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**First Derivative of parametric equations:** $$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \text{if } \frac{dx}{dt}\neq 0$$

**Second Derivative of parametric equations:** $$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} \qquad \text{if } \frac{dx}{dt}\neq 0$$

**Arc Length for parametric equations, for $\alpha<t<\beta$:** $$L=\int_{\alpha}^{\beta}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

**First Derivative of polar equations:** $$\frac{dy}{dx}=\frac{\frac{dr}{d\theta}\sin\theta+r\cos\theta}{\frac{dr}{d\theta}\cos\theta-r\sin\theta}$$

**Area of polar region**: $$A=\int_a^b\frac{1}{2}[f(\theta)]^2\,d\theta=\int_a^b\frac{1}{2}r^2\,d\theta$$

**Half-Angle Formulas**

$$\sin^2 x=\frac{1}{2}(1-\cos 2x)$$

$$\cos^2 x=\frac{1}{2}(1+\cos 2x)$$